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# UNIVERSAL PROPERTIES OF ANGULAR CORRELATIONS IN QCD JETS \*

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## Abstract

Predictions for angular correlations between an arbitrary number of partons are derived in the high energy limit. The quantities considered depend on angles and primary energy through a single variable  $\varepsilon$  which implies certain scaling properties and relations between quite different observables. These asymptotic predictions derived in the double log approximation of QCD are checked against Monte Carlo calculations at the parton and hadron level.

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## Abstract

Predictions for angular correlations between an arbitrary number of partons are derived in the high energy limit. The quantities considered depend on angles and primary energy through a single variable  $\varepsilon$  which implies certain scaling properties and relations between quite different observables. These asymptotic predictions derived in the double log approximation of QCD are checked against Monte Carlo calculations at the parton and hadron level.

## INTRODUCTION

Many new results from the detailed study of multiparticle production processes have been obtained in recent years, still there is no fully satisfactory model to describe all these phenomena. One approach is based on perturbative QCD, particularly suited for hard processes involving large momentum transfers, in which an initially scattered parton evolves by gluon Bremsstrahlung into a jet of partons and finally into the observable hadrons, whereby the effect of the color confinement force is modeled. Another approach relies on a statistical and thermodynamic description in which the microscopic degrees of freedom are integrated over and only global quantities are kept. Hadrons are produced from a quark gluon plasma after a phase transition. This approach is applied in particular to nuclear collisions in the search for the quark gluon plasma. As many aspects of particle production change only moderately when going from the more elementary collisions like  $e^+e^-$  annihilations to the more complex  $pp$  or nuclear collisions it is desirable to develop both the perturbative and the statistical methods and to explore their predictive power.

In any case, for a satisfactory model of multiparticle production we would like to have finally a mathematical model based on a simple principle, there should be only a few parameters, some results should be obtained analytically – even if only approximately – otherwise the structural properties of the theory can hardly be fully explored. A nice example of a model of this type is Hagedorn's bootstrap model: it is based on a single guiding principle, the bootstrap principle, the only parameters being the

particle masses and the interaction volume; analytical results on multiparticle observables can be derived from an equation for the generating functional of multiparticle densities.<sup>1</sup> Although it has been attempted to extend the statistical approach to hard processes<sup>2</sup> the most convincing results for such processes are obtained today from the parton cascade model as derived from perturbative QCD.

In this talk I would like to report on results from perturbative QCD on angular correlations inside a parton jet which are obtained in collaboration with Jacek Wosiek.<sup>3,4</sup> The essential parameters of the parton cascade are the scale parameter  $\Lambda$ , which determines the coupling strength and a cutoff  $Q_o$  which regulates the infrared and collinear singularities of the gluon Bremsstrahlung. As to the hadronization process we follow here the idea that it is sufficiently soft and the distribution of hadrons follows largely the distribution of partons.<sup>5</sup> In particular for momentum spectra of partons and hadrons such an equality up to a constant has been established, if the parton cascade is evolved down to the hadronic (i.e. pion) mass scale  $Q_o \approx m_h$ .<sup>6</sup> Such a similarity may also be expected if the observable considered is “infrared safe”, i.e. does not depend explicitly on the cutoff  $Q_o$ . It is one of the motivations of the present study, as to what extent such a hypothesis of soft hadronization is actually correct and supported by the experiment. It would allow for a certain class of observables to be calculated just only in terms of  $Q_o$  and  $\Lambda$  parameters, resulting in a scheme of high predictive power.

## THEORETICAL SCHEME

Our calculations are based on the double logarithmic approximation (DLA) of QCD. The probability to emit a single gluon of momentum  $K$  from a primary parton  $a$  of momentum  $P$  is given by

$$\mathcal{M}_{P,a}(K)d^3K = c_a a^2(K\Theta_{PK}) \frac{dK}{K} \frac{d\Theta_{PK}}{\Theta_{PK}} \frac{d\Phi_{PK}}{2\pi}, \quad (1)$$

where  $a^2 \equiv \gamma_0^2 = 6\alpha_s/\pi$  is the QCD anomalous dimension of the multiplicity evolution and  $c_g = 1$  or  $c_q = 4/9$  for gluon or quark jets respectively. For the running coupling constant we write  $a^2(p_T) = \beta^2/(\ln(p_T/Q_0) + \lambda)$  whereby  $\lambda = \ln(Q_0/\Lambda)$  and  $\beta^2 = 12(\frac{11}{3}N_c - \frac{2}{3}N_f)^{-1} \approx 1.565$  for 5 flavours  $N_f$ . In this approximation the recoil effects are neglected (i.e. energy and momentum conservation are violated), integrals are performed by retaining only the leading contributions from collinear and soft divergencies. The interference of the soft gluons is taken into account by the “angular ordering” prescription. In DLA one obtains the leading asymptotic behaviour, but the non-asymptotic corrections of relative order  $\sqrt{\alpha_s}$  are potentially large.

The generating functional which includes the “leading” primary parton in the final state is given by the non-linear integral equation<sup>7</sup>

$$Z_{P,a}\{u\} = u(P) \exp \left( \int_{\Gamma_P(K)} \mathcal{M}_{P,a}(K) [u(K)Z_{K,g}\{u\} - 1] d^3K \right). \quad (2)$$

In this form the virtual corrections are included which ensures the proper normalization  $Z_P\{u\}|_{u=1} = 1$ . The density distribution of  $n$  partons is then obtained by functional differentiation after the test functions  $u(k)$

$$\rho^{(n)}(k_1, \dots, k_n) = \delta^n Z\{u\} / \delta u(k_1) \dots \delta u(k_n) |_{u=1}. \quad (3)$$

Similarly, the cumulant (connected) correlation function is derived as in Eq.(3) but with  $Z$  replaced by  $\ln Z$ . Starting with  $n = 1$  one obtains from (3) and (2) linear integral

equations which can be solved recursively. Here we specialize on angular distributions and the corresponding equations are obtained after integration over the momenta. We have obtained results<sup>3,4</sup> on the general inclusive cumulant correlation function of  $n$  particles in their spherical angles. Of special interest are the distribution in the relative polar angle  $\vartheta_{12}$  between two partons and the multiplicity moments of general order  $n$  for particles falling into sidewise angular regions.

It turns out that all these angular observables can be constructed from a generic function  $h^{(n)}(\delta, \vartheta, P)$  which fulfils the integral equation

$$h^{(n)}(\delta, \vartheta, P) = d^{(n)}(\delta, \vartheta, P) + \int_{Q_0/\delta}^P \frac{dK}{K} \int_{\delta}^{\vartheta} \frac{d\Psi}{\Psi} a^2(p_T) h^{(n)}(\delta, \Psi, K). \quad (4)$$

where  $\delta$  and  $\vartheta$  are the small and the large angles of the respective problem and the inhomogeneous term behaves at high energies like  $d^{(n)} \sim \exp(2n\beta\sqrt{\ln(P\delta/\Lambda)})$ . One finds that the natural variables of the problem, instead of  $P, \vartheta$  are

$$\varepsilon = \ln(\vartheta/\delta)/\ln(P\vartheta/\Lambda), \quad \zeta = 1/(\beta\sqrt{\ln(P\vartheta/\Lambda)}). \quad (5)$$

As  $\vartheta > \delta \geq Q_0/P > \Lambda/P$ , we find  $0 \leq \varepsilon \leq 1$ . The solution for  $\ln h^{(n)}$  can be written as an expansion in  $\zeta \sim \sqrt{\alpha_s}$ . In the high energy limit ( $\varepsilon$  fixed,  $P \rightarrow \infty$ ) one obtains

$$h^{(n)}(\delta, \vartheta, P) \sim \exp(2\beta\sqrt{\ln(P\vartheta/\Lambda)}\omega(\varepsilon, n)). \quad (6)$$

The scaling function  $\omega(\varepsilon, n)$  is known in implicit form.<sup>3</sup> For small  $\varepsilon$  it has a power expansion  $\omega(\varepsilon, n) = n - (n^2 - 1)\varepsilon/2n + \dots$ . Another useful approximation obtains from an expansion in  $1/n$  which yields  $\omega(\varepsilon, n) \approx n\sqrt{1 - \varepsilon} (1 - \frac{1}{2n^2} \ln(1 - \varepsilon))$  and has a 1% accuracy for  $\varepsilon < 0.95$  already for  $n = 2$ . An asymptotic behaviour of type (6) was also found in the study of azimuthal particle correlations.<sup>8</sup>

## POLAR ANGLE CORRELATIONS

First we discuss the correlations  $\rho^{(2)}(\vartheta_{12}, P, \Theta) \equiv dn/d\vartheta_{12}$  in the relative polar angle  $\vartheta_{12}$  of two partons both inside a forward cone around the initial parton of half opening angle  $\Theta$ . For the study of scaling properties it is more convenient to consider the distribution in the variable  $\varepsilon = \ln(\Theta/\vartheta_{12})/\ln(P\Theta/\Lambda)$  given by  $\hat{\rho}^{(2)}(\varepsilon) \equiv dn/d\varepsilon = \vartheta_{12} \ln(P\Theta/\Lambda) \rho^{(2)}(\vartheta_{12})$ . For the correlation  $\hat{r}(\varepsilon) = \hat{\rho}^{(2)}(\varepsilon)/\bar{n}^2$  normalized by the multiplicity in the cone\*  $\bar{n}$  one obtains in the high energy limit ( $\varepsilon$  fixed,  $P \rightarrow \infty$ ) from Eq. (6) with  $h^{(2)}(\vartheta_{12}) \sim \vartheta_{12} \rho^{(2)}(\vartheta_{12})$  for either quark or gluon jet

$$\hat{r}(\varepsilon) = 2\beta\sqrt{\ln(P\Theta/\Lambda)} \exp\left(-2\beta\sqrt{\ln(P\Theta/\Lambda)}(2 - \omega(\varepsilon, 2))\right). \quad (7)$$

Differences between quark and gluon jets and the influence of the leading particle show up at finite energies where we obtain

$$\begin{aligned} \hat{r}_a(\varepsilon) &= c_a^{-1} y \exp(-y(2 - \omega(\varepsilon, 2))) - (c_a^{-1} - 1) y \exp(-2y(1 - \sqrt{1 - \varepsilon})) \\ &+ 2\beta\sqrt{2y/(c_a f(1 - \varepsilon)^{1/4})} \exp(-y(2 - \sqrt{1 - \varepsilon})) \end{aligned} \quad (8)$$

with  $y = 2\beta\sqrt{\ln(P\Theta/\Lambda)}$ ,  $f = 2\beta K_0(2\beta\sqrt{\lambda})/\sqrt{\pi} \approx 0.145$  for  $\lambda = \ln 2$ . To explore the scaling properties of  $\hat{r}(\varepsilon)$  we consider the quantity  $-\ln \hat{r}(\varepsilon)/(2\sqrt{\ln(P\Theta/\Lambda)})$  depending

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\*this normalization has better scaling properties<sup>4</sup> than the differential one used earlier<sup>3</sup>

only on  $\varepsilon$  for any momentum  $P$  or cone opening angle  $\Theta$ . The asymptotic limit and a finite energy result are shown in Fig. 1. As a test of our analytical calculations we also compared to the Monte Carlo evaluation of the parton cascade.<sup>†</sup> As can be seen from the figure, the predicted asymptotic scaling behaviour is nicely reproduced for small  $\varepsilon \leq 0.5$  in a large energy range ( $P \geq 20$  GeV) whereas deviations occur for larger  $\varepsilon$  (smaller relative angles  $\vartheta_{12}$  approaching the cut-off  $Q_0/P$ ). It should be noted that the normalization of the above expressions in (7, 8) and in particular the quark and gluon difference at finite energy are of non-leading order in the DLA and therefore less reliable. We have therefore adjusted the overall normalization of the functions in (7, 8) to the Monte Carlo data. We also studied the effect of hadronization provided by the HERWIG MC. Again these effects are negligible for small  $\varepsilon \leq 0.5$ .<sup>4</sup> Preliminary data from DELPHI<sup>10</sup> have recently given first evidence for approximate  $\varepsilon$ -scaling in  $\Theta$  ( $\Theta \geq 30^\circ$ ) and a close proximity of the data to the parton model results.

## MULTIPLICITY MOMENTS FOR SIDEWISE ANGULAR CONE OR RING

In a second application we consider particle multiplicities in a sidewise cone  $\delta\Omega$  of half opening  $\delta$  at polar angle  $\vartheta$  with respect to  $\vec{P}$  and in an angular ring of width  $2\delta$  symmetrically around the primary parton direction, centered at polar angle  $\vartheta$ . We will refer to the ring and the cone as the 1D and 2D configurations. We calculate the factorial and cumulant multiplicity moments  $f^{(n)}$  and  $c^{(n)}$  or normalized by the multiplicities  $\bar{n}$  in the respective angular regions  $F^{(n)} = f^{(n)}/\bar{n}^n$  and  $C^{(n)} = c^{(n)}/\bar{n}^n$  ( $f^{(2)} = \langle n(n-1) \rangle$ ,  $C^{(2)} = F^{(2)} - 1$ , etc.). The cumulant moments  $c^{(n)}$  can be derived again from the generic equation (4). For the normalized moments we find

$$C^{(n)}(\vartheta, \delta) \sim (\vartheta/\delta)^{D(n-1)} \exp\left(-2\beta\sqrt{\ln(P\vartheta/\Lambda)}(n - \omega(\varepsilon, n))\right) \quad (9)$$

with  $\varepsilon = \ln(\vartheta/\delta)/\ln(P\vartheta/\Lambda)$ . The dependence on  $\varepsilon$  is as in  $\hat{r}(\varepsilon)$  discussed above. Similar results have also been found by other groups.<sup>11,12</sup> In the modified LLA including next to leading effects in the exponent<sup>11</sup> the power changes typically by 10%.

The scaling properties of the various moments can again be conveniently investigated by projecting out the  $\varepsilon$ -dependence of the exponent in Eq.(9), one finds for

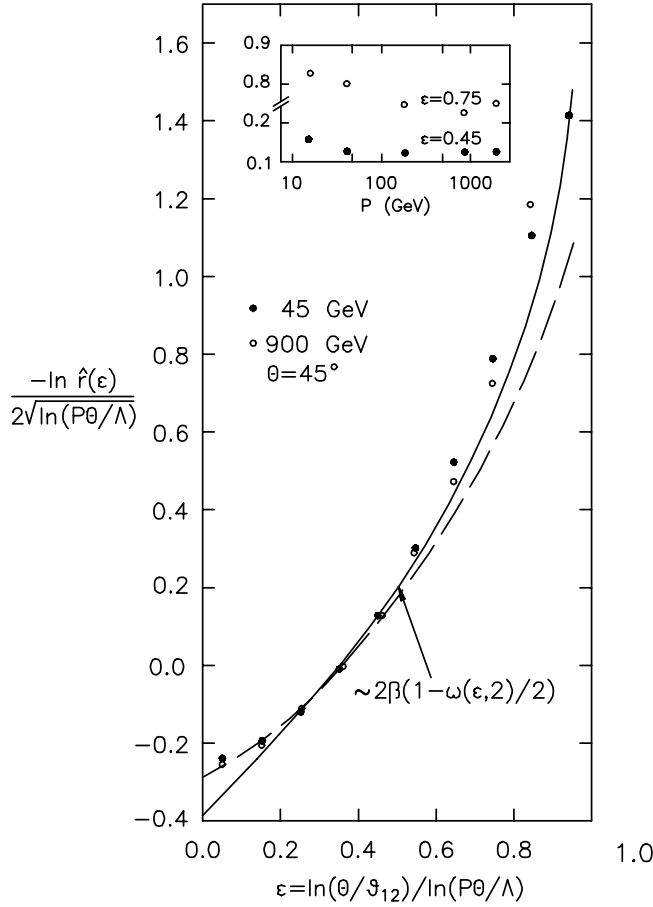
$$-\hat{C}^{(n)} \equiv -\frac{\ln[(\delta/\vartheta)^{D(n-1)}C^{(n)}]}{n\sqrt{\ln(P\vartheta/\Lambda)}} = 2\beta\left(1 - \frac{\omega(\varepsilon, n)}{n}\right) \quad (10)$$

in the high energy limit, or  $-\hat{C}^{(n)} \simeq 2\beta(1 - \sqrt{1 - \varepsilon})$  for large  $n$  independent of  $n, D$ .

In Fig. 2a we plot  $-\hat{C}^{(2)}$  for the ring ( $D = 1$ ) vs.  $\varepsilon$  for different primary momenta  $P$  for the parton MC. There is a violation of  $\varepsilon$ -scaling for small  $\varepsilon$  but the asymptotic prediction is approached for high energies. We have repeated the same calculation as in (10), but for the factorial moments  $F^{(n)}$  replacing  $C^{(n)}$  in (10), see Fig. 2b. As  $F^{(2)} = C^{(2)} + 1$  they approach the same asymptotic limit. Apparently, the nonasymptotic corrections are such that the scaling in  $\varepsilon$  for small  $\varepsilon$  sets in already at low energies for the factorial moments. The results for higher moments follow roughly the expectation (10), on the other hand the  $D = 2$  moments show a more gradual dependence on  $\varepsilon$  than predicted.<sup>4</sup>

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<sup>†</sup>we used the program HERWIG<sup>9</sup> with parameters  $\Lambda = 0.15$  GeV,  $m_q = m_g = 0.32$  GeV and without non-perturbative gluon splitting for the process  $e^+e^- \rightarrow u\bar{u}$ .



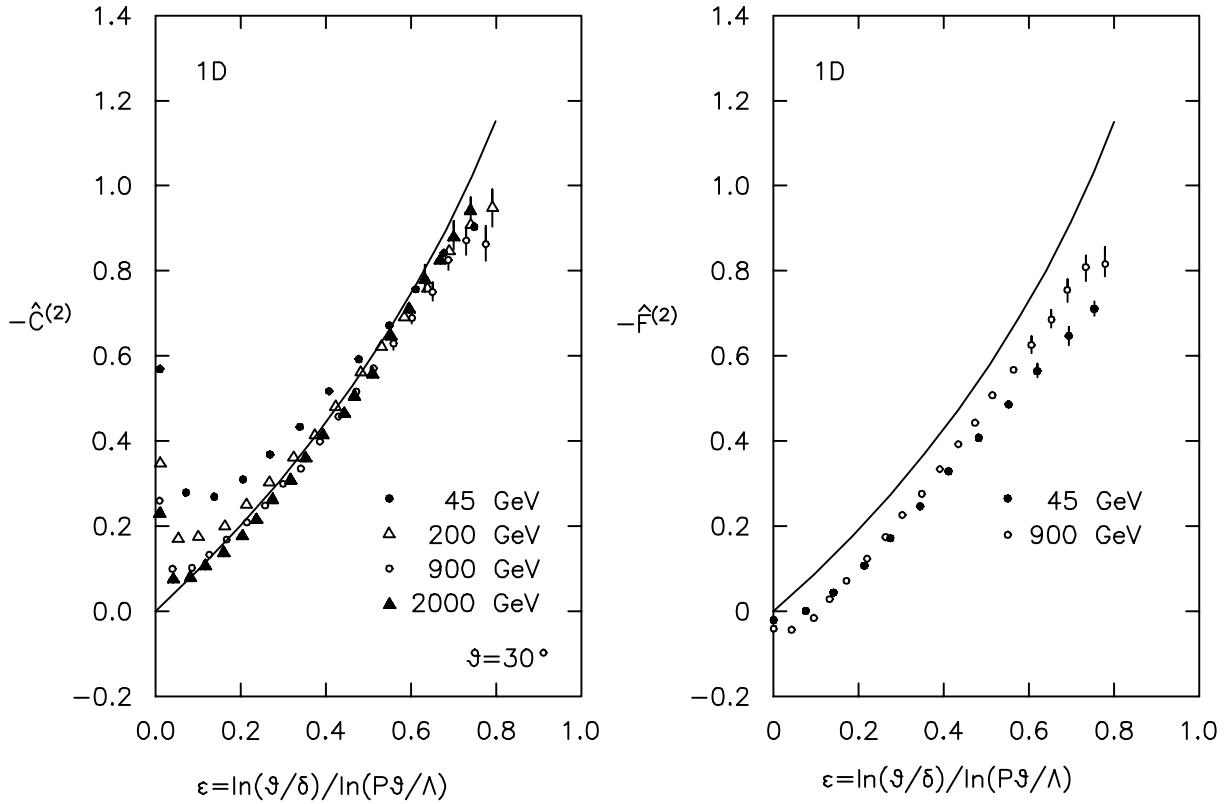
**Figure 1:** Rescaled polar angle correlation function for the high energy limit, Eq. (7) (full line), and for a quark jet of 45 GeV, Eq. (8) (dashed line), with normalization adjusted to the data as obtained from the HERWIG MC at the parton level. The insert shows the energy dependence of the same quantity for fixed  $\varepsilon$ .

An interesting aspect of these results is the remarkable universality of the various moments (different  $n, D$ ) and also of the very different observable  $\hat{r}(\varepsilon)$  which all converge against the same limiting function after appropriate rescaling (see Figs. 1,2).

## SUMMARY

The angular observables considered here, after appropriate rescaling, approach a limit in the normalized logarithmic angular variable  $\varepsilon = \ln(\vartheta/\delta)/\ln(P\vartheta/\Lambda)$  for  $\varepsilon$  fixed,  $P \rightarrow \infty$ . The comparison with the parton MC shows that this limit is already approached for 2-particle correlation  $\hat{r}(\varepsilon)$  and the factorial moments  $F^{(n)}$  at present energies sufficiently far away from the angular cutoff  $Q_0/P$  ( $\varepsilon \leq 0.5$ ) whereas the cumulant moments approach the asymptotic limit only at higher energies ( $P \sim 1$  TeV). In the region of small  $\varepsilon$  – not further discussed here – the observables approach a power behaviour in the angular variables and become independent of the cutoff  $Q_0$  (“infrared safe”). It is in this region that also the hadronisation corrections are found small.

The experimental confirmation of the universal high energy behaviour of rather different angular observables and of  $\varepsilon$ -scaling with two redundant variables  $P$  and  $\vartheta$  can provide further evidence for a soft confinement mechanism and parton hadron duality which allows to calculate hadronic distributions directly from the partonic ones.



**Figure 2:** (a) Rescaled cumulant moments for the ring as defined in Eq. (10) for the parton MC for different jet momenta  $P$  in comparison with the asymptotic prediction Eq. (10); (b) same as (a) but for factorial moments.

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